A.P. CALCULUS (BC)

Prerequisite: A grade of B or better in Calculus Honors or AP Calculus (AB) is strongly recommended
Meeting time: 5 days per week, full year, five credits
Placement: Grade 12, Level I

A.P. CALCULUS (AB)

Prerequisite: A grade of B or better in Advanced Mathematics (preferably Advanced Mathematics Honors) is strongly recommended
Meeting time: 5 days per week, full year, five credits
Placement: Grade 11, 12, Level I

A.P. Calculus (AB) is a college-level course in calculus and analytical geometry designed to prepare the student for the Advanced Placement Examination of the College Board, in order that they might receive credit and/or advanced standing at the college where they are to matriculate. The course assumes the student has a basic knowledge of elementary functions and trigonometry. These areas are covered in the honors section of Advanced Mathematics. The primary objective is to give a substantial experience in differential and integral calculus. Students will be required to complete summer assignments, which review important topics from previous math courses, prior to taking the course.

A.P. Calculus (BC) is a full year course in the calculus of functions of a single variable. Successful completion of our authorized AP Calculus (AB) course is a prerequisite. It is designed to prepare students for the BC Advanced Placement Examination of the College Board, in order that they might receive credit and/or advanced standing at the college where they are to matriculate. It includes a review of the more challenging topics covered in Calculus AB plus additional topics. Additional topics in this course include parametric, polar, and vector functions, and their derivatives, slope fields, L'Hopital's Rule, advanced integration topics, as well as various types of sequences and series, including the use of Taylor Series for approximation. The course is designed to be challenging and demanding, requiring the student to do a fair amount of independent work. Students will be required to complete summer assignments, which review important topics from the A.P. Calculus (AB) course, prior to taking the course.

(Agawam High School Academic Expectations: 1, 3, 4, 5, 6, 10)

COURSE GOALS AND OBJECTIVES

General Objectives

1. For A.P. Calculus (AB) and (BC), students will prepare to take the Advanced Placement Calculus (AB) or (BC) examination administered by the College Board in May

2. To provide the students with a challenging and stimulating college level mathematics course among peers who are both interested and talented in mathematics
3. To provide a solid mathematical background for students who intend to major in mathematics or science in college, as well as those who intend to major in a non-scientific field
4. To make the student sufficiently proficient in calculus so that he/she may successfully matriculate in an advanced calculus course at the college where they attend
5. To develop the student's ability to apply the concept of differentiation in solving problems
6. To generate in the student a recognition of, and an appreciation for, the beauty inherent in the logical structure of calculus
7. To develop, in the student, a thorough understanding of the concept of integration and its application
8. To provide the student opportunities for imaginative thinking in mathematics

Specific Objectives

1. To develop a thorough knowledge of the behavior of functions
2. To convey the concept of limits and their applications
3. To provide a solid foundation of the derivative of continuous functions
4. To develop a sound understanding of the first and second derivative tests and their use in determining the concavity of curves and maxima and minima values of continuous functions
5. To develop the ability to solve maximum and minimum word problems and related rate word problems with the use of differentiation
6. To convey an understanding of the Mean Value Theorem
7. To develop an understanding of the concepts of the indefinite and definite integral
8. To develop the ability to find the area under a curve and between curves
9. To provide a solid foundation in determining the volume and surface area of solids of revolution
10. To develop an understanding of differentiation and integration involving transcendental functions
11. To convey an understanding of more sophisticated techniques of integration

COURSE OUTLINE

The course outline below is adapted from the Advanced Placement Calculus Course Description booklet released May 2008 from the College Board. Items with a plus sign (+) in front of them denote additional topics covered in A.P. Calculus (BC).

I. Functions, Graphs, and Limits

Analysis of Graphs

Limits of Functions (including one-sided limits)

- An intuitive understanding of the limiting process
- Calculating limits using algebra
- Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior.
- Describing asymptotic behavior in terms of limits involving infinity
- Comparing relative magnitudes of functions and their rates of change

Continuity as a property of function

- An intuitive understanding of continuity
- Understanding continuity, in terms of limits
- Geometric understanding of graphs of continuous functions
- Intermediate Value Theorem
- Extreme Value Theorem

Parametric, polar, and vector functions

- (+) The analysis of planar curves includes those given in parametric form, polar form, and vector form

II. Derivatives

Concept of the derivative

- Derivatives presented graphically, numerically and analytically
- Derivatives interpreted as an instantaneous rate of change
- Derivatives defined as the limit of the difference quotient
- Relationship between differentiability and continuity

Derivative at a point

- Slope of a curve at a point
- Tangent line to a curve at a point and local linear approximation
- Instantaneous rate of change as the limit of average rate of change
- Approximate rate of change from graphs and tables of values

Derivative as a function

- Corresponding characteristics of graphs of f(*x*) and f'(*x*)
- Relationship between the increasing and decreasing behavior of f(*x*) and the sign of f'(*x*)
- The Mean Value Theorem and its geometric consequences
- Differential Equations

Second derivatives

- Corresponding characteristics of graphs of f(*x*), f'(*x*), f''(*x*)
- Relationship between the concavity of f(x) and sign of f''(x)
- Points of inflection as places where concavity changes

Applications of derivative

- Analysis of curves, including the notions of monotonicity and concavity
- Optimization of both absolute and relative extrema
- Modeling rates of change, including related rates problems
- Use of implicit differentiation to find the derivative of an inverse function
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations
- (+) Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration vectors
- (+) Numerical solution of differential equations using Euler's method
- (+) L'Hopital's Rule

Computation of derivatives

- Knowledge of derivatives of basic functions including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- Basic rules for the derivative of sums, products, and quotients of functions
- Chain rule and implicit differentiation
- (+) Derivatives of parametric, polar and vector functions

III. Integrals

Interpretations and properties of definite integrals

- Computation of Riemann sums using left, right, and midpoint evaluation points
- Definite integrals as a limit of Reimann sums over equal subdivisions

- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval

$$\int_a^b f'(x)dx = f(b) - f(a)$$

- Basic properties of the definite integral

Applications of Integrals

- Model physical, biological, or economic situations
- Using the integral of a rate of change to give accumulated change
- Setting up an approximation of Riemann sum
- Finding the area of a region, the volume of a solid with known cross sections
- The average value of a function
- The distance traveled by a particle along a line
- (+) Finding the area of a region bounded by polar curves
- (+) Finding the length of a curve in parametric form

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals
- Use of the Fundamental Theorem to represent a particular anti-derivative, and the analytical and graphical analysis of functions so defined

Techniques of antidifferentiation

- Antiderivatives following directly from derivatives of basic functions
- Antiderivatives by substitution of variables
- (+) Antiderivatives by integration by parts and simple partial fractions
- (+) Improper integrals

Applications of antidifferentiation.

- Finding specific antiderivatives using initial conditions, including applications to motion along a line
- Solving separable differential equations and using them in modeling. In particular, studying equation $y' = ky$ and exponential growth
- (+) Solving logistic differential equations and using them in modeling

Numerical approximations to definite integrals

- Riemann Sums
- Trapezoidal Rule

IV. (+) Polynomial Approximations and Series

Concept of Series

- (+) Sequence as a series of partial sums
- (+) Exploring convergence and divergence

Series of Constants

- (+) Motivating examples, including decimal expansion
- (+) Geometric series with applications
- (+) The harmonic series
- (+) Alternating series with error hound
- (+) Terms of series as areas of rectangles and their relationship to improper integrals, including the Integral test and its use in testing the convergence of p-series
- (+) The ratio test for convergence and divergence
- (+) Comparing series to test for convergence or divergence

Taylor Series

- (+) Taylor polynomial approximation with graphical demonstration of convergence
- (+) Maclaurin series and the general Taylor series centered at $x = a$
- (+) Maclaurin series for the functions e^x, $\sin x$, $\cos x$, and $\frac{1}{1-x}$
- (+) Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series
- (+) Functions defined by power series
- (+) Radius and interval of convergence of power series
- (+) Lagrange error bound for Taylor polynomials

RESOURCES

Primary Textbook:

Strauss, Monty J., Bradley, Gerald L. and Smith, Karl J. Calculus, 3rd Edition. Upper Saddle River, New Jersey; Prentice-Hall, Inc., 2002.

Secondary Texts:

Best, G & Fischbeck, S. AP Calculus with the TI-83 Graphing Calculator. Andover, Massachusetts: Venture Publishing, 1998.

Broadwin J, Lenchner G, & Rudolph M. Solutions: AP Calculus Problems Part II AB and BC. Bellmore, NY: Mathematical Olympiads for Elementary and Middle Schools, 2000

Computer Software:

Jackiw, N. (2001). *The Geometer's Sketchpad: Dynamic Geometry Software for Exploring Mathematics* [Computer program]. Emeryville, CA: Key Curriculum Press. (Version 4, Fall 2001)

ASSESSMENT STRATEGIES

- Class participation, written and oral communication
- Class work
- Homework
- Class projects and presentations
- Notebooks/Portfolios
- Quizzes
- Tests
- Semester Exams
- School Wide and Departmental Rubrics